Math 121

 $(29, 59.75, 74.5, 83, 90), \overline{x} = 70.3.$

1. (22 pts) (14, 15.5, 18, 18.75, 20), $\overline{x} = 17.4$.

Enter the list $\{21, 24, 25, 26, 29, 39, 41, 44, 48\}$ into the TI-83 and store it in list L_1 . Then select STAT > CALC > 1-Var Stats and press ENTER. Add L_1 to the command and press ENTER. The list of statistics appears.

- (a) The value of $\overline{\mathbf{x}}$ is the mean, 33.
- (b) The value of Sx is the sample standard deviation, 10.
- (c) The variance is the square of the standard deviation, so it is 100.
- (d) The z-score of 39 is $z = \frac{39 33}{10} = 0.6$.
- (e) The five-number summary: Minimum = 21, $Q_1 = 24.5$, Median = 29, $Q_3 = 42.5$, Maximum = 48.
- (f) The boxplot:



2. (4 pts) $(0, 4, 4, 4, 4), \overline{x} = 3.1.$

The mean is most likely to be greater than the median.

- 3. (10 pts) $(3, 7.25, 8, 10, 10), \overline{x} = 8.0.$
 - (a) For a uniform distribution, the height is the reciprocal of the base. So the height in this case is $\frac{1}{10}$.
 - (b) In this case, "greater than 6" means from 6 to 10. That interval has a width of 4, so the probability (using base × height) is $4 \times \frac{1}{10} = \frac{4}{10}$.
 - (c) The first quartile is 2.5. In terms of area, that separates the lower 25% from the upper 75%.
- 4. (8 pts) $(0, 1.25, 3, 8, 8), \overline{x} = 4.0.$
 - (a) The direction of extreme is to the left. The values to the left in the null distribution are less likely, while the values to the left in the alternative distribution are more likely.
 - (b) Find the area of the triangle with base from 0 to 1. The base runs $\frac{1}{5}$ of the way from 0 to 5, so the height is $\frac{1}{5}$ of the way from 0 to $\frac{2}{5}$. That would be $\frac{1}{5} \times \frac{2}{5} = \frac{2}{25}$. Now use $A = \frac{1}{2}bh = \frac{1}{2} \cdot 1 \cdot \frac{2}{25} = \frac{1}{25} = 0.04$.

- 5. (12 pts) (0, 9.25, 12, 12, 12), $\overline{x} = 10.4$.
 - (a) normalcdf(45,75,60,10) = 0.8664.
 - (b) normalcdf(-E99,50,60,10) = 0.1587.
 - (c) normalcdf(65, E99, 60, 10) = 0.3085.
 - (d) invNorm(.10,60,10) = 47.18.
- 6. (10 pts) (0, 5, 6, 8.75, 10), $\overline{x} = 6.5$.

The probability distribution of X is N(610, 25).

- (a) normalcdf(650,E99,610,25) = 0.0548.
- (b) To find the IQR, you must first find the 25th and the 75th percentiles and take the difference. So find invNorm(.25,610,25) = 593.1 and invNorm(.75,610,25) = 626.9. The IQR is the difference 626.9-593.1 = 33.8.
- 7. (6 pts) $(0, 2.25, 6, 6, 6), \overline{x} = 4.5.$

The Central Limit Theorem tells us that for large enough samples (50 is large enough), the distribution of \hat{p} is normal. That eliminates graphs (b) and (d).



The Central Limit Theorem also tells us that the mean of that distribution is p, which in this case is 0.30. That eliminates (a), which is centered at about 0.50.



That leaves (c) as the only possibility.



You could check further by computing the standard deviation of \hat{p} , which is

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.30)(0.70)}{50}} = 0.0648.$$

Three standard deviations is 0.1944, or nearly 0.20. From the center 0.30, the distribution in graph (c) extends from about 0.10 to about 0.50, which is consistent with the previous calculations.

- 8. (16 pts) (0.6.25, 10, 11, 15), $\overline{x} = 8.7$.
 - (a) According to the Central Limit Theorem, the mean of \hat{p} is p, which is 0.25.
 - (b) According to the Central Limit Theorem, the standard deviation of \hat{p} is

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.25)(0.75)}{2}} = 0.3062.$$

(c) The tree diagram:



Now summarize this in a table. If 2 people are in ICU, that is 100% of the sample. If it is 1 person, that is 50% of the sample. So the distribution of \hat{p} is

\hat{p}	$P(\hat{p})$
0.0	0.5625
0.5	0.3750
1.0	0.0625

- 9. (12 pts) $(0, 4.5, 9.5, 12, 12), \overline{x} = 7.7.$
 - (a) The distribution of \hat{p} is normal with mean p = 0.25 and standard deviation

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.25)(0.75)}{300}} = 0.025.$$

That is, \hat{p} is N(0.25, 0.025).

(b) The probability that \hat{p} is between 20% and 30% is normalcdf(.20,.30,.25,.025) = 0.9545.